The Arithmetic Teacher

APRIL 15 • 1954

The Arithmetic Curriculum—1954
CHARLOTTE JUNGE

Take-Away Is Not Enough
E. GLENADINE GIBB

Number Concepts for the Slow Learner
RUTH COOK

The How and Why of Discovery
ESTHER J. SWENSON

THE ARITHMETIC TEACHER

Published Quarterly by

The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

Associate Editor: ESTHER J. SWENSON, University of Alabama, University, Ala.

Subscription Price: $1.50 per year (four issues) to individual subscribers, $2.50 per year to others (libraries, schools, colleges, etc.)

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington 6, D. C.

OFFICERS OF THE NATIONAL COUNCIL:

President: John R. Mayor, Madison, Wis.
Executive Secretary: M. H. Ahrendt, Washington, D. C.
Past President: Harry W. Charlesworth, Denver, Colo.
Vice Presidents: Irene Sauble, Detroit, Mich.; Marie Wilcox, Indianapolis, Ind.; H. G. Ayre, Macomb, Ill.; Mary Rogers, Westfield, N. J.

DIRECTORS: William A. Gager, Gainesville, Fla.; Lucy E. Hall, Wichita; Kan.; Houston T. Karnes, Baton Rouge, La.; Harold P. Fawcett, Columbus, Ohio; Howard F. Fehr, New York, N. Y.; Philip S. Jones, Ann Arbor, Mich.; Elizabeth Roudebush, Seattle, Wash. Secretary to Board of Directors: Agnes Herbert, Baltimore, Md.

The National Council has for its object the advancement of mathematics teaching at all levels of instruction. Any person who is interested in the field of mathematics is invited to apply for membership. Teachers should include the name and address of their school on their application. The membership fee is $3 per year and this entitles each member to receive *The Mathematics Teacher*, the official journal, monthly except in June, July, August, and September.

Table of Contents

		Page
The Arithmetic Curriculum—1954	*Charlotte Junge*	1
Take-Away Is Not Enough!	*E. Glenadine Gibb*	7
Number Concepts for the Slow Learner	*Ruth Cook*	11
The How and Why of Discovery in Arithmetic	*Esther J. Swenson*	15
A Selected Bibliography of Research in the Teaching of Arithmetic	*E. Glenadine Gibb*	20
Bibliography for Teachers	*Editors*	23
History of Mathematics in Teaching Arithmetic	*Margaret F. Willerding*	24

THE ARITHMETIC TEACHER

Volume I Number 2

April 1954

The Arithmetic Curriculum—1954

CHARLOTTE JUNGE
Wayne University, Detroit, Mich.

THE MODERN ARITHMETIC CURRICULUM is an outgrowth of various influences which have been operative over a period of years. Some factors have been in the picture for more than thirty years, others are of more recent origin—or are being recognized more recently as having significance. Among the more prominent of these influences are the changing philosophy of education, increased knowledge of child development, clarification of the nature of human learning, a rethinking of the purposes of instruction in arithmetic, and changes in our culture. As a result of these influences arithmetic curriculums are becoming much more realistic about what children need in arithmetic and what seems appropriate and within children's capabilities at various age levels. Considerable research evidence has been accumulated on the nature of the learning process in arithmetic, the nature and the place of meanings in arithmetic, and upon the comparative value of various methods of instruction. These researches have significance for the teacher and curriculum maker, and although the research is not as complete as is to be desired, every advantage should be taken of the findings thus far in planning sound progress in arithmetic for children of elementary school age.

Trends in the Organization of the Curriculum

The psychology of arithmetic, as indicated by recent research, stresses the point of view that arithmetic should not be viewed as a number of discrete, unrelated elements to be taught incidentally or through repetitive drill, but rather as a closely knit system of ideas, principles, and processes. Accordingly the real test of learning is not facility in figuring, but in an intelligent grasp of number relations and the ability to apply these into quantitative situations with proper comprehension of both their mathematical and practical significance. Instruction in arithmetic aims at developing meaning and understanding and in helping the child see sense in what he does. This means teaching arithmetic with attention to the number system, for meaning in arithmetic inheres in the system itself.

The meanings in the number system are dependent, one upon the other. The simple, fundamental meanings are acquired first and the more complicated ones grow from these. The instruction of the learner must be continuously related and interrelated in larger and more significant patterns. This makes organization in the teaching of arithmetic an inescapable necessity.

Now, what does this mean for the curriculum maker in arithmetic? It means just this—that if arithmetic is to be taught with meaning and understanding as basic goals, then it must be taught with attention to the number system and to the relationships which exist within that system. This means a continuous course, a systematically organized course for even the first grade. It means that basic concepts must be thoroughly developed first so that the superstructure of facts and processes may be built upon them later.

The term "systematic instruction" should not be misunderstood. It does not mean formal or abstract. It means simple, systematic, and regular learning provided for in the schedule. Arithmetic is a subject which more than any other subject in the curriculum is organized and sequential. Accomplish what you can through incidental teaching, but never let it be supposed that in first grade or any other grade, a satisfactory course in arithmetic can be provided without being planned and systematically pursued.

Furthermore, the research in learning indicates that all learning is dependent upon previous learning and serves as a basis for future learning. Each new concept which the child meets in arithmetic is interpreted by him in terms of his previous experiences. Modern arithmetic curriculums recognize the importance of sequence in learning and are so planned that what a child learns today leads easily and naturally into what he learns tomorrow, the learnings at one grade level merge carefully with those of the next level, and basic principles continue as ever expanding threads of understanding throughout the whole of arithmetic.

It is interesting to note that present curriculums are as concerned with the sequential development of the "intangibles" in arithmetic as they are with the development of concepts and understandings. Many good programs give emphasis to the development of desirable habits and attitudes about arithmetic, to the steady development of ways of thinking about quantitative situations, to giving children progressively more mature ways of analyzing and solving problem situations. Growth in habits of neatness and accuracy of recording, of persistence in seeking solutions, in the use of initiative and the development of independence are stressed for it is recognized that such things develop as a result of planned instruction just as much as an understanding of the multiplication facts.

Arithmetic, when regarded as a system of quantitative thinking is probably one of the most complicated subjects which children have to face in the elementary school. It is difficult because it is abstract. A great mistake was made in earlier arithmetic curriculums in rushing children too rapidly from one idea to another, in moving too rapidly from concrete situations to abstract applications. One of the basic principles which governs the construction of arithmetic curriculums today is that learning is a slow process and a relatively long time should elapse between a child's first introduction to a process and his mastery over it. Time must be allowed for meanings to develop before the children are expected to employ them in highly habituated reactions. In programs aimed at the development of understandings, *time is one of the dimensions of learning.*

In the learning of any concept or skill there are various degrees of understanding. Not all relationships can be learned at one time, and then, too, the significance of such relationships depends, in part upon the experiences which accompany acquisition of these relationships. Curriculum builders have recognized this by spreading the learning of any one concept over a wide range of time instead of concentrating it all at one level. Consequently, skills and processes formerly taught at later levels in the elementary school are being extended over a longer period of time. For example, basic concepts which relate to multiplication and division, the ideas of equal groups, appear early in the work of first grade but no formal teaching of these concepts takes place until the third and fourth grades, and long division with two and three place divisors is the work of the fifth grade. The child in kindergarten and first grade is provided with many experiences which deal directly with ideas of measurement and fractions, the basic concepts are expanded in third and fourth grades, and the more difficult aspects of computation are taught in fifth and sixth grades.

This spreading of topics in arithmetic over several grades is a direct reaction to earlier programs which had a tendency to move arithmetic into the upper grades of the elementary school, thus throwing a heavy burden on both the teacher and the children of these grades. One of the main outcomes of the present emphasis on meaningful arithmetic has been this im-

provement in gradation of arithmetic materials and the development of programs which recognize the fact that maturation of ideas, skills, and concepts requires instructional programs so organized as to contribute to the steady development of understanding of and insight into mathematical relationships as the child advances from level to level in the school.

Closely related to this curriculum plan which spreads learning over several grades is the concept of readiness. Readiness to learn is an important consideration in all learning at all levels. The concept of readiness was formerly considered to be synonymous to mental maturity and related to grade placement. However, it is now recognized that readiness in arithmetic is a function not only of mental maturity but also of previous experiences, methods of learning, interests, attitudes, and purposes. Readiiness is the concern of all teachers at all grade levels. It is not something to be waited for but something to be developed in the child by carefully planned learning experiences. Furthermore, the level at which a child may profit from opportunities to learn varies in the case of individual performances. A child is ready for different kinds of learning at different ages.

Reflection of this readiness concept is found in present day curriculums in various ways. First of all, readiness to begin arithmetic is encouraged by providing many wide experiences with number in the kindergarten and first grade and an environment so stimulating that there will be a constant need for the use of number and quantitative ideas. Little pencil and paper work is done at this level and the emphasis in instruction is upon discussion, the use of real materials, and the solution of quantitative situations, which arise normally in the classroom, through the use of concrete and pictorial materials. These programs reflect the idea that readiness for arithmetic is built into the individual best by a process of rich, first-hand experiences, plus reflective thinking about these experiences. The quality of the concepts developed in this way, the degree to which they are understood, is dependent entirely upon the *quality* of the first hand experiences and the quality of the thinking about those experiences.

Furthermore, present curriculums recognize that readiness is a consideration throughout all of the arithmetic in the elementary school, and to this end they provide readiness activities in connection with all new concepts in the curriculum. For example, activities which build a readiness for multiplication are incorporated in textbooks and curriculums long before the children begin to work systematically in this process. Exercises which build a background for understanding the addition of fractions, computation with decimals, and the use of formulas are woven into the curriculum long in advance of the actual work on them. Thus the children proceed smoothly from one step to the next in learning. Recently a sixth grade boy commented to me about his teacher. "I like the way Miss S. teaches. She just doesn't throw things at you, she leads up to them gradually so they aren't hard when you get to them!" Obviously, Miss S. was a teacher who built the readiness background carefully before undertaking a new concept with the children.

Undoubtedly one of the most difficult and perplexing problems faced by curriculum makers in arithmetic is the wide range of differences among individuals in any one class. Psychology indicates that children, within a class differ one from the other in mental ability, in the rates at which they learn, in interests, in the kinds of difficulties they encounter, in their responsiveness to corrective and remedial measures, as well as in their achievement. Psychologists also point out that while children differ in their rates of progress and their accomplishments at any one time, they also show a high degree of similarity in the order in which different developments appear and are consolidated. The total picture is one of growth rather than one of unique stages and differences are a matter of degree only.

In an attempt to meet these individual differences, curriculums today include suggestions and materials to be used in challenging the gifted child in arithmetic as well as recommendations for types of re-teaching activities which may be used

with the child who is achieving below the level of his class mates. In addition to this a study of curriculums and textbooks (which often constitute the curriculum in a school) reveals a tendency to keep children in any one class moving forward on an even front. This is all the children in the group work on the same skill or process, but at different levels. One finds keyed exercises which refer children having difficulties to simpler exercises on the same topic while children who have achieved understanding carry on activities at a more advanced level. For example, in a class working on the subtraction of fractions, some children may be working with concrete materials, others may be using diagrams and recording solutions with numbers, and still others may be working at the abstract level solving problems involving this skill. All children are working on the same process, but each is proceeding at his own level of thinking.

The preceding paragraphs indicate that curriculums today recognize the importance of vertical integration in the development of understandings in arithmetic. It should be pointed out, that the importance of horizontal integration is also recognized. While most educators agree that arithmetic must be taught as a separate subject, they also agree that it is most important that arithmetic be integrated with the other areas in the curriculum. Consequently, curriculums and textbooks carry many activities and suggestions of ways that arithmetic may be made to function in the social studies and science, in music and physical education, in reading and the other language arts. It is pointed out that many times the comprehension of materials in these other areas is dependent upon an understanding of quantitative terms and concepts. Upon occasion materials from the social studies or science may be used during the arithmetic period to introduce new concepts, to provide practice on concepts already learned, or as a source of problem material in connection with the concepts being learned. Similarly, curriculum guides suggest that eachers of the other curricular areas take time to develop the meanings of the quantitative ideas wherever they occur. Horn[1] in discussing this topic says, "The frequency and difficulty of the demands made upon arithmetic in attacking problems in other fields suggests that instruction in arithmetic can make important contributions to facilitating the work in these fields." He also states, "no one can seriously question the fact that a considerable amount of arithmetic is learned in connection with the study of problems in other areas. Indeed if the arithmetical demands in other fields are met, marked contribution from these fields is inevitable not only to the motivation for learning arithmetical abilities but also to their development and maintenance."

Trends in Content of Curriculums

The overall objective of arithmetic is to teach children to think quantitatively and to help them apply this ability in the solution of the problems of daily life. This objective has definite implications for the content in arithmetic. First of all, it indicates that arithmetic curriculums will not only stress the ability to compute but will also include an emphasis on the number system and number relationships. It also indicates that these skills and understandings will be developed best if they are presented in situations which are real in the lives of children.

A study of curriculums developed within the last five to ten years reflect these two emphases. Almost without exception the content of arithmetic guides and textbooks include content aimed at developing an understanding of the number system, the four processes with whole numbers and with fractions, and the rationale of arithmetic. Research has indicated that curriculums and methods of teaching which emphasize rational procedures appear to be the most economical route to speedy and accurate computation. The research also indicates that learning acquired in this way is more lasting and that it is more functional—children are better able to apply it in the solution of problems in new situations.

This emphasis upon the development of

[1] Horn, Ernest. "Arithmetic in the Elementary School Curriculum." The National Society for the Study of Education, *Fiftieth Yearbook*, Part II. 1951, pp. 10–11.

meaning and insight is as much a part of the curriculum of grades one and two as it is of the upper grades. Or, perhaps, it should be stated the other way around! When the meaning theory of teaching first made its appearance in the literature, there was a tendency to develop understandings of the simple, basic processes with children, but to resort to "telling" when teaching the more complicated skills. This latter situation seldom exists at the present time. Meanings are consistently taught from grades one through six in the elementary school with no break at upper grade levels.

Educators, in preparing curriculum materials for the elementary grades draw heavily upon a knowledge of the life and needs of children in these grades and upon the extensive research concerning children's interests and activities. They try to insure that the content will be genuine and lifelike and that the children using the curriculum materials will come to see arithmetic as a very normal part of their everyday life. The problem content in today's arithmetic classes relates to the type of situations familiar to children at that age level—to the games they play, to allowances and to saving, to buying and selling, to travel and to sports. Children are helped to solve arithmetic situations which arise in their home and school life and to collect and discuss current materials which show the application of arithmetic in the world today.

In addition to this many books and curriculums include items designed to help children learn something of the history of arithmetic, of the number system, of weights and of measures. It is important that children see that arithmetic is not only a science, but also a cultural subject with a long history, and to understand something of the contributions made by great civilizations to our present day arithmetic. This attention to the historical aspects of arithmetic is commonly found in curriculums in third grade and above.

Closely related to this is the inclusion in some curriculums of appreciation units which not only provide activities which will help children see the important role of arithmetic in society, in sciences, and in industry but which also include number games, puzzles, number tricks, and magic squares. It is plain to be seen that arithmetic is becoming a much broader and more interesting subject than it was formerly.

There are two areas of arithmetic content which have been brought more sharply into focus in the most recent school programs, and which are important enough in the total program to deserve special mention. First of all, greater attention is being given in all grades to development of abilities in problem-solving. Verbal problems are included as a part of the work at all grade levels—even the first, and effective problem-solving helps are systematically provided. These problem-solving activities aim at helping the child understand and see the relationships between what he wants to find out and the known facts. They seek to help the child develop his own way of solving problems and to leave him with a method of attack on quantitative situations. Consequently, modern programs in arithmetic encourage the use of problems to introduce new concepts, for practice on concepts which have been developed and for evaluation and testing of concepts learned. Problem-solving is assuming a role of major importance in programs based on the development of meanings.

The second area of content receiving special attention at the present time is that of mental arithmetic. In all grades, from grade one on, attention is given to oral arithmetic, to mental computation, to the development of the ability to estimate, to round numbers, to judge the reasonableness of answers and to use standard reference units in interpreting quantitative statements. Research indicates that most computations in daily life are done without pencil and paper and that if children are to use arithmetic effectively in their daily life then they must learn to compute mentally. Many programs include a well-organized program in estimating, judging the reasonableness of answers and oral arithmetic. Indications are that consistent attention to these skills develops a keen number sense, a healthy self-reliance, and the power to think with numbers.

Summary

In this discussion an attempt has been made to discuss some of the more significant trends in the organization and content of arithmetic curriculums today, and to show the relationship of these trends to research in education and psychology. The research in these areas must form the basis for any significant reform in curriculum and instruction in arithmetic. We are long past the time in educational method when rule of thumb and uncontrolled speculation are adequate for improving the learning process. Fundamental advances in the quality of instruction at any level in any school will depend upon the careful translation of the results of research into specific working ideas for method and curriculum.

THEN AND NOW

In Adam's New Arithmetic published at Keene, N. H. in 1838 are these problems.

"A cucumber grew to the length 1/3960 of a mile; what part is that of a foot?"

"If 11/13 of a pound of sugar cost 7/15 of a shilling, what will 32/43 of a pound cost?"

These problems were written to use numbers and processes for their own sake and not for functional usefulness. They "trained the mind" according to an older concept of faculty psychology. Do you suppose someone had a tape or ruler scaled so he could measure such fractions of a mile? Has anyone ever seen a scale graduated in 13ths and 43rds of a pound? Although we have discarded faculty psychology we have not cast aside experience in analysis and thinking. However, it is possible to analyze and to think with useful operations in a setting that has significance to the learner.

Today the ordinary intelligent citizen needs much more arithmetic and mathematics than he did 125 years ago. Many new situations and concepts have become a part of our ordinary experience. Consider calories, compression ratios, 5-10-5, income tax, installment buying, etc. But these are not always computational situations and in many cases an intelligent understanding and response should be made without paper and pencil. We must learn to sense, to think with, and to form valid conclusions in terms of the mathematics in a situation. Let us be conscious of the large role played by concepts, information, and mathematical principles as well as computations.

FUN WITH NUMBERS

Magic Squares

A magic square is an arrangement of numbers in a square like that on the lower left. It is "magic" when each row, each column, and each principal diagonal produce the same sum. How many different arrangements of the digits 1–9 can be placed in the 3 by 3 square? How should the 4 by 4 square be completed using numbers 1–16?

6	1	8
7	5	3
2	9	4

4	5	11	14

Sectioning Quantities

1. How can the water in an 8-gallon container be divided into two equal amounts by using only the original container and one each of 5 and 3 gallon capacities?
2. It is said that it is possible to weigh on a balance scale any integral quantity up to 40 pounds by using only four different weights? What are these weights and how must they be used for every weight from 1–40 pounds?

Take-Away Is Not Enough!

E. Glenadine Gibb
Iowa State Teachers College, Cedar Falls

"There were 8 children playing at Mary's house. Then 2 children went home. How many children are still at Mary's house?"

"Don has 5 balloons and 2 whistles. How many more balloons than whistles does he have?"

"Lynn needs 4 more apples to fill a box of 8 apples. How many apples were already in the box?"

"Now," says Miss Jones, "to solve these problems you just subtract."

A silence falls over the second grade class. Shadows of perplexity darken the children's faces. Immediately, Miss Jones attempts to remove this doubt by saying, "See, it is very easy. You just take 2 from 8. You take 2 from 5. You take 4 from 8."

Another pause—and then Johnny speaks for the group. "It may seem easy to you," he says, "but it is *certainly hard* for us."

Yes, some teachers and some textbooks tell the second grade child that these problems are "subtraction problems," or they simply say that "to find the answer you subtract." Yet, a comment as made by Johnny elicits these questions: (1) Do these problems really illustrate the same type of situation for the child who is learning to recognize subtractive situations? (2) Is the teacher merely providing a chaotic learning experience for her class as she glibly tells them that these are subtraction problems?

Classifying the kinds of situations the child meets in the primary grades may help us to see what difficulties beset the child who is learning arithmetic. In this paper three classifications of problem situations have been made. They have been referred to as subtractive problems, additive problems, and comparative problems, respectively. Identification of these situations in which the processes of subtraction and addition are applicable in determining a solution follow. Each has been illustrated with a problem as commonly found in number books for use by primary grades.

I. Subtractive problems. *When a sub-group is removed from a given group* (a sub-group is a part of a group, that is, 4 and 2 are each sub-groups of 6), *then*

a. If a sub-group is removed from a given group, one can find how much is left; as,
   Six birds were in a tree
   Two of the birds flew away. (Subtractive—"left")
   How many birds were left in the tree?

b. If an unknown sub-group is removed from a given group and the remainder is known, one can find how much was taken; as,
   Dick had 25 cents in his pocket.
   After spending some money, he had 11 cents left. (Subtractive—"taken")
   How much money did he spend?

c. If the sub-group is known and the remainder is known, one can find the original group; as,
   Jim took 3 books from the table.
   There were then 6 books left on the table. (Subtractive—"group")
   How many books had been on the table?

II. Additive problems. *When two groups are combined, then*

a. If two known groups are combined then one can find the total group; as,
   Bill had 35 cents. (Additive—"how many in all")
   He earned 40 cents. Then he had just enough to buy a game.
   How much did the game cost?

b. If an unknown group is combined with a known group to give a total which is also known, one can find the unknown group; as,
Mary had 4 cents.
She wants to buy a 15-cent puzzle. (Additive—"more needed")
How much more does she need?

c. If a known group is combined with an unknown group and the result is known, one can find how many there were in the original group; as,
Jane had some cookies on a plate.
After she put 4 more cookies on a plate, there were 9 cookies. (Additive—"how many had")
How many cookies had already been on the plate?

III. Comparative problems. *When a comparison is made between two groups, then*

a. If the number of objects in each group is known, one can find how many more objects are in one group than in the other; as,
Tom sold 10 tickets for the class show.
Jerry sold 7 tickets. (Comparative—"more")
How many more tickets did Tom sell than Jerry?

b. If the number of objects in each group are known, one can find how many fewer objects are in one group than in the other; as,
Betty bought a book for 15 cents.
Phil bought a book for 25 cents. (Comparative—"less")
How much less did Betty pay for her book than Phil paid for his book?

It becomes the responsibility of the primary-grade teacher to provide experiences for children in such a way that they may perceive the use of the processes of subtraction and addition in solving these kinds of problems For example, let us consider a problem illustrating one aspect from each of the three general kinds of situations described above. (I) A child has 8 blocks on the table. To find how many are left after he takes 5 blocks away, he sees he can solve the problem by taking 5 blocks from the 8 blocks, or $8-5=3$. Or, (II) he has 5 blocks on the table. He wants to find how many more he needs to have 8 blocks. He sees he needs to add more blocks to get the 8 blocks and may proceed to add the blocks. Yet, he is told that he is to solve this problem by taking away the number he has from the number he wants, or $8-5=3$. Now, let us examine (III). He not only has a group of 8 blocks on the table but he also has another group of 5 blocks. This situation may be changed so that he has a group of 8 blocks and a group of 5 boxes. He is asked to find how

many more blocks in one group than in the other, or how many more blocks than boxes? Although he sees 13 distinct objects before him (as illustrated above), he must see that to find how much more one group is than the other, he takes from the larger group the number of objects necessary to match with each object of the smaller group, and his answer is then that which remains.

From an examination of some materials used by children in the primary grades, one finds that children are expected to solve all of these kinds of situations consecutively—at least all these problems appear on one page or on consecutive pages in textbook and workbook materials. On the other hand, one also finds other textbooks and workbooks which provide much opportunity for working in one kind of situation before using the same process to solve another kind of problem situation.

An apparent "trouble-spot" for young children and the diversity of thought on the part of adults in providing materials to give experiences in these kinds of problem-solving situations gave direction for a study of children's thinking as they solved some problems similar to those described above. The purpose of this study[1] was to

[1] A summary of this study, "Children's Thinking in the Process of Subtraction," Unpublished Ph.D. thesis of the writer, University of Wisconsin Library, will appear in a forthcoming issue of *The Journal of Experimental Education*.

analyze the processes of a group of second grade children used in solving selected problems. From the analysis made of those processes, an attempt was then made to answer the following questions: (1) Do applications of subtraction present situations in which young children respond differently? (2) Do the kinds of materials used in presenting the problem influence the children's responses to the problems?

Nine problems were given to each of 36 second grade children randomly selected from 24 state-graded schools in Dane County, Wisconsin. Three subtractive problems were given to find how much was left after some had been taken away. One of these problems was given with toys or other manipulative materials. Another problem was given about a group of circles (or squares) mounted on a card. A third problem was a written description of a problem situation. Following the placing of the material before the child, the problem that he was to solve regarding it was given verbally or read to him. Similarly, three additive problems were given to find how many more should be added to a given group to form a desired group, and three comparative problems were given to find how much more one group was than the other.

Some of the implications for problem solving in the primary grades suggested by this study follow statements of findings given below.

Regarding applications of subtraction:

1) *Of three situations studied, subtractive problems (Type Ia) were easiest to understand, took less time, were most readily associated with the idea of taking one group from another, and were easiest for obtaining a correct result.* This implies that children respond to the reality of the problem. When taught the process of subtraction as a take-away process (and these children had been taught that idea), it is much easier to use it in solving problems which are directly solved by taking a group away. Furthermore, this kind of problem presents a simpler situation to the child and thus should be used before other kinds of situations, as described above, in developing his ability to solve problems.

2) *Of three situations studied, additive problems (Type IIa) were solved more commonly as applications of the process of addition rather than as applications of the process of subtraction.* Again, there is the implication that children solve problems in terms of the reality of the problem. Thus, one may well expect a child to solve these problems by addition as $5+__=8$ and to think of them as addition problems. One may also expect children to become confused if expected to think immediately of them as subtraction problems. Carefully planned experiences and maturity in problem-solving are necessary if a child is to make the generalization that these are applications of subtraction.

3) *Of three situations studied, comparative problems (Type IIIa) were hardest for these children to understand. Approximately as many comparison problems were solved by the process of addition as by the process of subtraction. It was more difficult to give correct solutions to comparative problems than to the other kinds of problems.* These findings question some organizational sequences where comparative problems follow subtractive problems and come before additive problems as an order of relative difficulty for children. Although comparative problems may be more closely perceived as problems solved by subtraction, other aspects of the problems would indicate that they are more difficult to solve. In these problems a child is often confronted with unlike groups of objects. For example, if asked to tell how many more blocks than boxes, he may be expected to respond by giving the number of blocks, for, the boxes are not blocks and should thus be left out.

4) *There were no noted differences among these three applications for the way in which the problems were solved (counting, number facts, etc.) or the amount of verbalization necessary to explain the method of solution. Verbalization seemed most directly related to individual differences.* A child may be expected to use whatever level of grouping he has reached as a means of finding the solution to any problem. If a child is only able to count one object at a time, one may expect him to solve his problems in that manner regardless of the difficulty of

the situation. If a child recognizes groups without counting each object or uses number facts, then he may be expected to use those methods in reaching his solution. The length of the explanations children give in describing their solutions would imply that some children are just inclined to talk more than others.

From the three findings above one may expect only a confused group of children if these kinds of problems are given without regard to kinds of learning difficulties involved. Certainly, these results show the need for well-organized and carefully planned classroom experiences over a long period of time. From such experiences children may then be able to make the generalization that there is a process (subtraction) which solves five of these problem-solving situations.

Regarding materials used in presenting problems:

1) *Children more readily understood the problems, solved them by the process of subtraction, obtained correct solutions, and worked them in less time when the problems were presented with circles (or squares) on cards. They did not do as well with the problems when given the toys or other manipulative objects and had most difficulty with problems presented with a written description.* Possibly problems presented to children in real-life situations and those presented with representations in geometric forms do not present similar difficulties. The toys, whether in reality or in picture, may be expected to suggest other interests to the child. This suggests that teachers must provide for such social embedding which is found in the problems a child solves in real-life situations. Other kinds of manipulative materials or picture materials might have given different results. However, either of these kinds of materials (toys and circles) directly associated with the problems are more helpful to children in solving problems than those problems which are described in words. This would suggest that the use of much written problem-solving material be discouraged in the primary grades.

2) *The process which was used and the verbalization in explaining the process were not substantially altered by change in materials used in presenting the problems.* One may expect the level of counting which a child has reached (one-to-one correspondence, recognition of groups, number facts, etc.) to be independent of the materials used in the problem. Again, the kinds of materials used may not be expected to encourage a child to talk more about this means of solving the problems.

Regarding other observations which the analysis made possible:

1) *There was no evidence of a general subtraction ability.* This would not support any statement which would imply that either a child can or cannot subtract.

2) *Children did not respond equally well to problems presented with similar materials.* One cannot expect similar values to be gained by all children from the same experience, that is, the same experience is not equally appropriate for all children.

3) *No score gave a uniform picture of the child's level of response to a problem.* The evaluation of one aspect of behavior does not represent the total behavior in making the response. Thus, it would be desirable for teachers to evaluate the process of solving a problem as well as to evaluate the final outcome. This suggests that looking only for a right or wrong answer gives a very limited picture of a child's ability to solve problems.

These implications are based on results of an investigation of the problem-solving behaviors of a group of children in one locality. Findings for other types of localities are needed to support these results, and in turn, to support the implications for teaching suggested by the results of this study.

Number Concepts for the Slow Learner[1]

RUTH COOK
Phoenix, Arizona

THE CHILDREN in our cerebral palsy classes are not slow learners in the usual sense of the word suggesting mental deficiency, but rather they are slow learners because their natural intelligence cannot function normally as a result of their brain injuries. Their brain injuries are responsible not only for their motor handicaps, but auditory, visual, and speech handicaps as well. Many of them also suffer a sensory loss in spatial relationships; they are unable to estimate distance and often cannot count unless they can touch the objects they are counting. Of the twenty-eight children we have in two classes, ten are nonambulatory, twenty-four have motor handicaps of varying degrees, seventeen have speech defects, two have rather severe hearing loss, eleven have visual defects, four have convulsions, and about fifteen are left-handed. This averages about three handicaps per child.

Brain injured children are not all concentrated into classes such as the ones we conduct at our Easter Seal Center; there are possibly brain injured children in every regular classroom throughout the land who are usually labeled as morons, feeble-minded, or slow learners. Maybe they are the "fidgety" children. Most of these children suffer from short attention span so that it practically requires magic tricks to keep them interested in one subject for a reasonable time. This distractibility is such that if some child leaves his place, or if there should be a sudden noise, his attention is gone. Most brain injured children are incapable of sustained effort.

These are two ways of coping with this distractibility. One is to control as much as possible the external, over-stimulating environment, at the same time developing the attractiveness of the subject being taught. A second and more difficult way of handling distractibility is to educate and help the child in developing voluntary control; to prolong gradually his attention span.

A classroom of brain injured children should necessarily be small, not exceeding twelve in number. The reason for keeping these special classes small is understandable, for, where in a so-called normal situation one has as many personalities as there are children, in a class of brain injured children, since no two have identical handicaps, their differences are accentuated out of proportion.

Now consider a class of these children with varying types and degrees of handicap; some don't walk, some have no speech, most have poor hand coordination, auditory troubles or an imbalance of eye musculature and control, and about half of them have some sensory loss. Their perception and reception is faulty and they are unable to produce the way they want to, the thoughts that they think. From earliest babyhood on, their experiences have been limited. Most of them never have had the pleasure of exploring and getting into things, which is learning by doing. Naturally, their maturational age is far behind that of the so-called normal child.

How can number concepts be learned by children whose experiences have been so limited? Where most babies learn about objects by reaching, grasping and releasing, some of our children are unable to do even that. Next the baby learns by pointing to objects, but with a severe sensory loss plus poor coordination, some of our children have not arrived at that stage. Later the growing child makes mental observations by simply looking, but where imbalance of coordination extends to the eyes also, it is most difficult for these

[1] Presented at the meeting of the National Council of Teachers of Mathematics at Los Angeles, December, 1953.

children to look down a column of figures without touching each number at the same time.

In spite of all their differences, these children are still more like normal children than they are different, so we provide a curriculum as nearly like that in regular grades as possible, but individualized to suit each pupil's capacities and limitations, with particular emphasis on a prolonged readiness program.

Some educators have recommended putting off the teaching of arithmetic until about the third grade, and concentrate first on the language arts. At first thought, it sounds like a reasonable suggestion, particularly for the cerebral palsied brain injured children. Of course, if arithmetic is a matter of memorizing formulas and drill, we would agree, but at our school we consider mathematics as a language art. One of the most characteristic things about mathematics is the language used in expressing ideas of quantity and relationships. No language is more clear and exact than measurements of time and weight and space. Try to imagine communication without words such as tall, short, deep, early, now, yesterday, heavy.

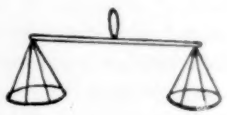
Mathematics is also a social study because its measurements of distance, time and weight are social necessities developed over many centuries of experience and discoveries. In presenting this heritage to children, it seems practical to take them back to the simple beginning of the simplest ideas, to rediscover the first concepts, so that they might be meaningful to the children and thus they form their own concepts.

To do this, we bring as many practical experiments into the classroom as possible. For example, children soon learn to tell by the clock when it is recess time and lunch time and time to go home. We talk about other ways that have been used to tell time. The children were greatly interested in making a shadow clock on the playground; a crude sort of sun-dial.

Each morning one child takes his turn at pinning up the date on our large calendar with blank spaces. We count the weeks and days until some child's birthday, or a holiday such as Christmas.

Before considering weights and scales, we notice the balance of the teeter-totter. We experiment with heavier and lighter children, and note the difference when they change positions. Someone balances a stick on his finger and notices that the place where it touches his finger when balanced, looks like the middle of the stick. A balancing see-saw is built of tinker-toys with two lids taped to the extremities, and the children balance and weigh lumps of modeling clay. Now we are ready for one of our Josh and Jake stories.

Jake is a shepherd who has a flock of sheep, and Josh is a farmer who raises beans. Jake has a big family of children who are tired of eating mutton three times a day, so Jake trades one of his sheep for a stone weight of Josh's beans. Probably the first scale was Jake's shepherd staff balanced from the center over the crotch of a tree. It is natural to suppose that the first standard of weight was a stone. (Even today a little English boy might weigh three stones.) So with Jake's staff having a stone fastened to one end, and a leather bag full of beans at the other, a fair trade was enacted. The following day, those children who are able, make scales with a stick, string, and two paper plates.



Even those who are unable to participate to any helpful extent might hold the materials in readiness for those who are doing the work. Then we bring into the classroom a scale borrowed from the therapy room. All the children are weighed and a chart is made of their weights to be compared with future weighings.

In measuring, we might first use just an ordinary stick. We find that Laura is two sticks high and a little bit more. Dick is taller, being almost three sticks high. It is story time now, so we have one about Josh using a measuring stick to measure his bean poles. He needs tall poles for the bean plants to grow on. There is no sense in his lugging big poles around to try to get them the same length, when a little stick laid out three times is so much easier.

After this sort of introduction, our yardstick and ruler can be fun.

On some days we play with water. It is another experience in which Cerebral Palsy children have been cheated. But while we play with water, we also measure with measuring cups, fruit jars, and quart bottles. The children estimate how many times one can empty a pint jar into a quart jar, and some of the guesses are wild of course, though most of them are fairly accurate. Someone wonders if a fruit jar holds as much as a quart milk bottle; most of the children think the milk bottle must hold more because it is taller, so comparisons are made and they form their own conclusions. An experience chart at the end of our water play goes like this:

A quart is big.
A pint is little.
A half pint is half as much as a pint.
One cup is the same as a half pint.
A quart jar is two times as big as a pint jar.
Four cups fill the quart jar.

The children play store, post office, and bank. They make their own price tags, use real money, have sales, etc. No matter how realistic one makes these classroom social situations, they are of rather dubious value unless one also takes the children on field trips to see and have the experiences in true situations. The classroom experiences are merely a dramatization, but they help to confirm the true concept of social necessities.

As a science, mathematics is an art in which children learn to estimate, compare, find errors, make corrections, check, and verify. It is a science requiring logical and precision-clear thinking. It can never be half right.

We have found no way of presenting numbers to children so scientifically correct, and at the same time so much fun, as Catherine Stern's Structural Arithmetic. It is created at the child's level, using the child's own vocabulary in a series of situations and games from which the desired concepts emerge naturally. With these concrete teaching devices, children gain an insight into the value and relationships of abstract numbers, and the structure of our decimal system. Where other methods of concrete materials have to do with counting, structural arithmetic is based on measuring. It is a method of discovery. The child sees the number relationships, his hands are busy arranging the numbers, and his mind is busy thinking, thinking numbers. He can easily see any error he makes and do his own correcting, thus making every part of his learning meaningful. When a child picks up a 6-block, he knows that it comes after the 5 and before the 7 in the number series. He knows that it is one more than 5 and one less than 7. He knows that it will take the 4-block added to the 6-block to be as much as the 10-block. He knows that it holds sixth place in the number series and that it is composed of 6 ones.

Perhaps one of the most difficult of number hurdles is to explain the true meaning of the tens. We tell the children how our Arizona Apache Indians carry with them a leather pouch with pebbles in it to represent the number of ponies they own, except that when the row of pebbles reaches way out it gets harder to count. Now Jake, "away back when," thought up a smart idea when he had to keep track of how many sheep he had in his flock. He scratched two lines on the ground and laid out his pebbles on the line to the left. When the number of pebbles reached 10, he removed the 10 pebbles and placed 1 pebble in a line on the left. Thus 1 pebble counted for 10. Then he would continue his counting. We illustrate with the abacus to show that it tells the same story as Jake's marks and pebbles, but again Catherine Stern's dual board shows plainly what takes place when 10 units are converted into one 10-block, and it is easier for the children to understand because it gives a truer picture of what actually happens.

To show further development of the number system, another story is in order to explain how it all might have started in the first place. Jake's flock of sheep had grown so big that he divided them among his sons to take out to pasture in different directions. When cold weather came, they brought their sheep together to the winter pasture in the valley. Of course they were anxious to know how big their flocks had

grown with all the baby lambs that had been born. Jake squatted on the ground and kept count with his pebbles as the sheep passed single file through the gate. The two sons, standing on either side of the gate, checked on their father's counting by using a different method. The older son counted with his fingers as the sheep went through the gate. As soon as his tenth finger went up, he nodded his head to his brother, who put up one finger which stood for one ten. The flocks had grown so large that the second son soon used up all his fingers.

"Father," he called, "what shall I do? I've used up all my fingers, and the sheep still come. There are more than 10 tens."

"Put out your foot, son," replied Jake, "that will be one hundred, and then start all over again. I'll scratch a third line for my pebble counting."

How they love to dramatize this story; the sheep in wheel chairs and on crutches, going around and around the room through an improvised gate made of two chairs. Jake sits on the floor counting with pebbles, and the sons perform with their fingers.

This, then, is our readiness program: As a language art to make clear all types of comparison; as a social study with necessary experiences the children need to know about in everyday life pertaining to distance, weight, size, speed, time, cost, quantity, and form; and as a science using pure numbers and number relationships, getting a true concept of our number system. This threefold program seems a meaningful way of presenting mathematics, not only to brain injured children whose lack of experience makes it necessary, but to so-called normal children as well. The needs of our cerebral palsied children are the same as those of all children, except perhaps a little more so.

BOOK REVIEW

How Big? How Many? Arithmetic for Home and School, Gladys Risden, Boston, Christopher Publishing House, 1951, Cloth, 248 pages, $3.50.

In this book Dr. Risden, who is a child psychologist and former teacher, sets forth an array of experiences in teaching arithmethic meaningfully. In a brief introduction she draws the line between "two kinds of arithmetic:

(1) The quantitative questions of life that demand thinking of the combination and separation of quantities.

(2) The problems in the book that demand remembering the rule and Teacher's explanation."

The main body of the book is then presented as a series of anecdotal records in each of which some phase of the "first kind" of arithmetic is taught. The episodes have their settings variously in home and classroom, and in all of them children are being encouraged by teachers, by parents, and by each other to "think out" answers to arithmetic problems which need solving. A considerable variety of problems is thus dealt with, from elementary addition facts to decimal fractions, per cent, and measurement.

This book should be of special interest to those teachers of arithmetic who, while they favor completely the teaching of arithmetic from a meaningful approach, do not know quite how or where to start (probably because they, themselves, learned by a rule-memorize-drill routine). In the first place, it is completely nontechnical, and requires no special training in mathematics to read and understand; rather, the author writes with simplicity and clarity, and with a fine gift for evoking mental images of the situation she is describing. Furthermore, many of the anecdotes can be made almost immediately applicable to classroom situations; as a whole they are sufficiently diverse and sufficiently numerous to suggest to a thinking teacher many similar experiences as possible learning contexts. This is the author's objective to begin with.

REVIEWED BY ROBERT L. SLAUGH

The How and Why of Discovery in Arithmetic

ESTHER J. SWENSON
University of Alabama

THE TRAVELER had occasion to drive frequently on the main highway between two large cities. There were very few towns on the 100 miles of highway and he found himself breaking the monotony by noticing highway signs and other details which told him what places were located along the road in each direction.

One sign interested him particularly. It was one of a cluster of signs pointing off to his right as he drove from his home. The road indicated was an ordinary gravel road which crossed a railroad running parallel to the highway, then disappeared quickly into a national forest. The sign said simply, "Payne Lake"—nothing more, no mileage indicated. It could just as well have said "Jones Lake" or "Long Lake" or "Forest Lake"; the word which interested The Traveler was "Lake." Being a native of a state with many lakes, he was always attracted to them, particularly while living in "a far country" where lakes were a rarity instead of the ever-present feature of the landscape which he had known as a child.

The first time The Traveler saw the sign, he was interested but did not have time to leave the highway to investigate the whereabouts of Payne Lake. Every time thereafter that he passed the sign he wanted to follow it; but, not knowing the distance or anything else about the trip, he did not investigate the intriguing road into the woods.

The next summer he drove the same road with a friend as they started out on a leisurely weekend trip. Just before they reached the sign, The Traveler told his friend about how much he would like to go in search of Payne Lake. He asked if the friend would object to making a detour of unknown length over an unknown road to a practically unknown destination just to satisfy The Traveler's curiosity. Being a good sport, the friend encouraged him to start out. They agreed that they would go no more than 10 miles; if they didn't get to Payne Lake within that distance, they would turn back.

This had been a hot Saturday at the end of a hot week of summer school! In the woods they caught a delightfully cool breeze every now and then. The road was a little bumpy but not bad enough to discourage them. They just had to drive more slowly than they had been doing on the paved highway. The road led up hill and down dale with many twists and turns. Time and again when they reached the top of a hill, a breath-taking mountain forest view would make them drive still more slowly so as to take it in more adequately. The time went fast even though they drove slowly; very quickly, it seemed, the speedometer told them that they had driven 10 miles. They had seen no sign of a lake. What should they do? They certainly did not want to turn back, so they just revised their original agreement. They said they would drive at least 2 more miles unless they came to another sign sooner than that. They did come to another sign at a crossroads; it told them to turn right but gave no mileage. The Traveler and his companion agreed to go up to 15 miles from the highway. They were having too good a time to turn back, and Payne Lake *might* be just over the next ridge. They certainly wouldn't want to come so far and then just barely miss it, especially since they had been reassured that they were on the right road.

No, 15 miles of driving did not bring them to Payne Lake; but another sign, this time to the left, lured them on. At 17 miles they actually reached Payne Lake. It was a beautiful gem of a lake. The long leaf pines grew close up to the shore all along its irregular points and inlets. The sunlight sparkled on the gently waving pine needles almost as brightly as it did on

the lake. The Traveler got a real thrill out of that journey's end. He felt as if he were looking at a pine-fringed lake in his native state.

He was Columbus! Payne Lake was America! He had discovered Payne Lake! Oh, yes, he has learned more about the place since then. He has visited it frequently enough to know where the good picnic spots are, how one makes arrangements with the game warden to fish there, how the lake looks from the top of the nearest fire tower, and where one can find the largest pine cones for one's open fireplace. His enthusiasm for the place led him to ask several people in his home city if they had been there. Most of them said, "No." Each time he got such a response, he felt so much more the discoverer. The psychologists would say he had "identified" himself with Payne Lake as its discoverer.

Of course, The Traveler would have been quite willing to concede that there were earlier discoverers, some so early that they chose the site on which Payne Lake was to come into being. The CCC boys who helped build the dam and lay out the camp shelters and work on the road were there long before he was. The conservation officials who planned it all must have felt as if they were not only the discoverers but the creators of Payne Lake. No matter! So far as he was concerned, The Traveler and his friend discovered Payne Lake on a hot day in August when road signs without mileages lured them on that voyage of discovery.

This little account of the discovery of Payne Lake may serve as a parable to represent some of the whys and hows of discovery in the learning of arithmetic. With this story as our text, let us see what applications we can find in it to use in the teaching of arithmetic.

A child learning arithmetic (or anything else, for that matter) is often a traveler in a new and strange country. He looks about him for landmarks, for ways to become oriented, to "know where he is." As he travels along the smooth highway of arithmetic, he may see interesting by-paths. Whether the signs indicating "places to go" in arithmetic appeal to him or not depends a great deal upon his past experiences and how the signs to new paths of endeavor tie in with these experiences.

Why did the word "lake" interest our Traveler so much? Because through many, many experiences on, in, and connected with lakes he had built up a rich background of meanings for the word. A lake to him had become a symbol for fun—fishing, swimming, camping. It had become a symbol of beauty—rippling waves under a bright summer sun, frozen expanses of whiteness in winter, sandy beaches, the cool green of trees along the shore, sunrise and sunset reflected across the water. It had become a symbol of his "home country," where lakes were so much a part of the meaning of another word—"home."

When a child starts out on a new venture in arithmetic, he cannot have an adult's lifetime store of experiences as his background; but he can and should have a backlog of number experiences which is as rich as possible for his present maturity level. Sometimes he has had the number experiences but needs help in relating them to one another and to the task at hand. Sometimes he has had experiences with arithmetic which have been so confusing and frustrating that his experience-backlog is a hindrance, not a help. Some people wouldn't drive a single mile off the road to see a lake, particularly an unknown lake. Some people—both children and adults—wouldn't exert themselves one bit to seek an experience with number; many would exert a great deal of energy in avoiding any experience associated ever so slightly with arithmetic. It is not only the amount of previous experience a child has had with arithmetic that matters, but also the quality of the experience he has had.

But, you ask, what if the children I teach do not have a backlog of rich and satisfying experience in the field of number relationships? The answer is simple to state but very complex in its execution: Help them to build up that experience. Notice the statement does *not* say *"Give* them experience"; we cannot give experience to another. We can only help him

to get his own experience. We cannot go into the experience-building program of elementary school arithmetic fully here. Suffice it to say that such a program is an integral part of any successful attempt to emphasize discovery in arithmetic. Children cannot be expected to "go exploring" in arithmetic until they have a "home territory" of number meanings which is adequate as a "home base" for the particular excursion into the unknown.

Our story tells us that The Traveler passed the Payne Lake sign many times before the day when his curiosity triumphed. Two factors were different on that occasion from those which had been in operation on previous occasions. First, he had always been in a big hurry to get somewhere within a limited time. That day he was not hurried; he did not feel "pushed." Second, on that day of discovery someone else encouraged him to follow the lead of the Payne Lake sign.

Wouldn't it be interesting to know how many times in the elementary school life of the average child he is deterred from following his curiosity about number relationships, from exploring the intriguing by-paths which he does see—to say nothing about those he has not been helped to see—all because someone was in a hurry? Maybe he himself is in a hurry to get through with his arithmetic assignment so he can do something he enjoys more. Maybe his teacher is in a hurry to get through the book by the end of the year or to finish a certain chapter before six weeks' examinations. It is not enough to admit that our educational haste makes much educational waste. We ought to do something about it. We ought to quit hurrying long enough to see the opportunities for learning right at the side of the road we travel every day. We get ourselves into the ridiculous situation of hurrying through the course of study so fast that the children we supposedly teach are forced to lag on the road to fundamental learnings. Instead of helping them on their way, we set up road blocks to hinder their progress.

His friend encouraged The Traveler to follow the lead that aroused his curiosity. Children in school need to be encouraged to follow up their ideas, their clues to solutions. We often penalize a child because he wants to solve a numerical problem situation by going off on what we consider a "tangent." Children do need guidance in problem solving; that's what teachers are for. But they do not need the restrictive type of guidance which says, "This is the road I always follow; you'll be wrong to follow any other." Encourage the boy or girl who has a bright idea of a road to an original solution. It may be better than any you know or any given in the text. Some cases of curiosity are not easily discouraged; many are. Most of us "believe in signs" enough to believe that a highway marker pointing in a certain direction means that the place indicated is really to be found in the direction shown. In our guidance of children's learning, we should check on the confidence we can probably place in the clues to be followed. Given that reasonable confidence, a large part of our job is to encourage learners to follow the clue.

Because the Payne Lake road signs did not give the mileage, our travelers did not know when they started out whether they would go the whole distance or not. Accordingly they set up intermediate goals for themselves—they would go 10 miles, they would go 2 more miles, they would go 15 miles, and so on. When we set out to establish arithmetical relationships, to develop generalizations, to solve problems, the goal is not in full view; if it were, there could be no element of discovery at the end of the journey. If the destination is quickly or very easily reached—say, if Payne Lake had been only a mile off the highway—intermediate goals are not of great importance; if the arrival at the destination is delayed, intermediate goals are tremendously important. As each is approached or realized, one is encouraged to go on to the next. In general, the less mature the learners, the more dependent they are upon the spacing of intermediate goals which keep up their interest and feeling of "getting somewhere." The skillful teacher learns to judge this situation and to help children plan accordingly.

Some advocates of learner activity, learner responsibility, and dependence

upon the learner's own initiative and creativity give the impression that good teaching is a matter of turning everything over to the children and trusting to their innate inventiveness to carry them through to a worthwhile resolution of all difficulties. This situation would be analogous to that which would have prevailed if someone had said to The Traveler, "Somewhere in this county you can find Payne Lake. See if you can find it." That kind of so-called "challenge" would not have challenged him at all; it is too free, too uncontrolled to be of much assistance to a person. He found Payne Lake without an undue expenditure of time and trouble because there were road signs all along the way pointing the direction. Every time he saw one, he knew he was still on the right track. Sometimes the "road signs" for the child who is seeking to find a new arithmetical relationship turn up as part of his exploring activities. He gets a partial answer which fits with his original clue and encourages him to continue his search. Sometimes he does not see that he is making progress; the teacher needs to step in at that point and help him to see what has been accomplished (where he is now) and to plan next steps (where to go from here and in what direction).

The road to Payne Lake was bumpy in places and full of twists and turns—not as smooth traveling as the main highway but not so bad as to outweigh interest in reaching the objective. So also is the road of discovery in learning arithmetic. We, that is, the teacher and the pupils, cannot safely drive at breakneck speed on this road. We must slow down enough to stay on the road if we wish to prevent the trip from becoming too discouraging and uncomfortable. So handled, the bumps are soon forgotten and the twists and turns become part of the challenge, for who knows around which bend or after what slow climbing ascent will be found the goal or another look ahead, a new view bringing new satisfaction and new understanding. The road to the thrill of discovery in arithmetic has on it many "high points" and thrilling "views ahead."

At last, sometimes at long last, we do arrive at our big Point of Discovery. There it is—our destination! We didn't know in advance what it was going to be like exactly. We had some ideas or we would never have started out to find it. This is where the pieces fit together. What we find is ours. We found it ourselves. The teacher helps the learners along the way—helps them organize their thinking about the goal, helps them evaluate their progress, helps them see whether or not they seem to be on the right track. But when they solve the problem, when they can put the generalization into their own words, when the relationships slip into place, at that point the learner feels that *he* did it. It is *his* discovery. And it will remain his. He is proud of it, and the chances are pretty good that he will want to go on more journeys of discovery, mathematically and otherwise.

Some readers may question this article's use of the term "discovery." "Dis-cover" means to un-cover. The solution may have been available all the time; others may have uncovered it long ago and made it theirs; but to this learner who is making the discovery, it is uncovered for the first time. The relationships were present but he has to see them for himself; he has to remove the misunderstandings or lack of understanding which has previously hidden the solution or the relationships. He may have had help, but not too much help. He must have been encouraged to exert enough of himself in the solution that it is his own—not that someone else can say, "You did it yourself" but that he can honestly tell himself, "I figured it out myself. I did it."

The original story of the trip to Payne Lake ended with arrival there. Of course, once there The Traveler and his companion had to think how they should get back on the highway. They could go back the way they had come; that would mean, they knew, that their total detour would amount to 34 miles. At the lake, however, they saw another sign which told them that by going on around the lake they could get to a town called Centerville. Again, no mileage was indicated. They knew that their original route on the highway went through Centerville. In the confident flush of past success, they de-

cided to continue their explorations. The road did lead them back to their main route of travel and with less driving then would have been necessary had they returned by the way they came. They saw more new country and got oriented geographically in more ways than before. The trip of discovery, which started out as a following of by-paths, led the travelers back to the main road. Would that more teachers were less afraid of promising by-paths, and that more children could have the thrill of traveling those by-paths to significant learnings, always coming back to the main highways of education bringing with them new meanings, new understandings, and new discoveries which are not so easily found on some of our rigidly controlled and monotonously managed and maintained roads of instruction. If the main road is the closest of all roads to the fundamental needs of learners, it deserves to be called the main road and we had better not wander too far away from it. But if the "main road" is so-called just because it is the most commonly traveled road, where the guides do the same old things in the same old way just because that is the line of least resistance, we might all—teachers and learners alike—have more fun and end up knowing more arithmetic if we follow fruitful by-paths to discovery.

At first, The Traveler was puzzled to find so few people in his home city who knew about Payne Lake. It seemed to him like a case of ignoring the Acres of Diamonds in one's own back yard. It seemed so sad that people were ignoring the opportunity for the enjoyment of Payne Lake's recreational offerings. Then, one day, just by chance he heard someone talking about Lake Margaret. He inquired concerning its location. Lo and behold! It was in the same location as Payne Lake! As a matter of fact, he learned that Lake Margaret had been recently renamed Payne Lake. He realized at once that some of those people who didn't know *his* "Payne Lake" probably did know Lake Margaret, and perhaps knew it better than he did.

In arithmetic, too, we sometimes depend too much on *our* terminology for a discovery. Perhaps if we were willing to talk things over with other teachers and with our pupils, we might discover more agreements than we suspected. It is so easy to get in the habit of expressing our discoveries in certain words and of expecting children to use *our* words for *their* discoveries. Such behavior on our part not only bars us from understanding the progress children are making toward arithmetical discoveries but may also confuse them so much that they may never discover what they otherwise would have discovered.

Finally, let us return to the tragedy of The Traveler's neighbors who didn't know Payne Lake by that or any other name. Let us consider them as representing people who have missed out on the thrill of discovery in arithmetic. We may well wonder how many of our past arithmetic pupils have missed that experience.

How many such do you have in your pedagogical past? How many such are there among the elementary school population of the U. S.? Too many! Far too many! That admission should lead us to effect some changes in arithmetic teaching so that the future may have in it a much higher proportion of pupils who *have* been helped to use their inquiring minds on the road to discovery in arithmetic.

ARITHMETIC OR ELEMENTARY SCHOOL MATHEMATICS

An increasing number of schools are calling their work *Elementary School Mathematics* instead of *Arithmetic*. They do this to connote a broader content than the "skills and problems" which typified many programs of arithmetic. This same proposition arose when THE ARITHMETIC TEACHER was christened. In this *journal* the two names will be used synonymously to denote a rich body of materials growing out of the quantitative relationships required of an intelligent citizen.

A Selected Bibliography of Research in the Teaching of Arithmetic

E. Glenadine Gibb
Iowa State Teachers College, Cedar Falls

This annotated bibliography consists of selected studies reported in the literature within the last ten years by teachers who have carried on research in their own classrooms (as nearly as the writer has been able to ascertain) and by others who have used research techniques which are applicable to ordinary classroom situations.

Teachers in elementary schools often seek guidance for more effective ways of providing learning experiences in the classroom. Commonly, they make decisions on the basis of subjective recall of previous experiences with other classes, or simply by intuition. However, there are teachers who have made an effort to make such decisions from a research approach. In so doing the generalizations which each teacher has been able to make about a group of children (or a child) may then serve her, and only her, as a guide in making decisions involving other boys and girls whom she has not yet taught. For example, generalizations which Miss Smith might be able to make from a study she has made in her fifth grade class are applicable to future fifth grade classes which Miss Smith has not yet taught. Such generalizations are not necessarily applicable to Miss Brown's fifth grade class across the hall or to Miss Jones' fifth grade class in another school. It might be noted here, by way of comparison, that studies in what is commonly known as fundamental educational research are so designed that the researcher may know to what extent his generalizations are applicable to other "fifth grades as well as to future fifth grades." No attempt has been made in this summary to identify the extent to which generalizations can be made from the findings of the studies included here.

The following bibliography does indicate some of the answers teachers have obtained from their investigations, and some of the possibilities for other studies which might be made by classroom teachers, enabling them to make more effective decisions with regard to their practices in the teaching of arithmetic.

1. Clark, Eileen, "Number Experiences of Three-Year Olds," *Childhood Education* 26: 247–250, February, 1950.

   Reports a brief study of young children's use of numerical terms. Suggestions were made in accordance with findings regarding ways the nursery school could stimulate and further interest and comprehension of number.

2. Cruickshank, William M., "Arithmetic Ability of Mentally Retarded Children, I. Ability to Differentiate Extraneous Materials From Needed Arithmetical Facts," *Journal of Educational Research* 42: 161–170, November, 1948.

   Reports an investigation made of normal and mentally retarded boys with regard to their ability to select specific arithmetic elements needed in the solution of problems. Found a greater difference between the two groups in solving problems from which nonessential elements had been eliminated than in problems where subjects were asked to identify unneeded elements. Normal children were more adept in differentiating unneeded material and more successful in working problems freed from extraneous material than were the mentally retarded group.

3. Davis, Robert A. and Rood, Edward J., "Remembering and Forgetting Arithmetic Abilities," *Journal of Educational Psychology* 38: 216–222, April, 1947.

   Study made to determine the extent to which pupils retain certain basic skills

while studying increasingly complex material and thus acquiring new related knowledge. Findings suggest need for integration between courses on various grade levels to provide opportunity for using previous learning.

4. Harvey, Lois F., "Improving Arithmetic Skills by Testing and Reteaching," *Elementary School Journal* 53: 402–409, March, 1953.

Reports a study of the effectiveness of diagnostic tests and remedial instruction in helping children improve their skill in arithmetic computation. Presents an analysis of errors made in multiplication with description of method of teaching used. Found procedures used a good method for helping most children improve their skill in multiplication.

5. Hirsch, Martin, "Does Changing the Form of a Problem Affect Its Difficulty?" *High Points* 33: 19–25, December, 1951.

Reports a study made of 3 examples involving division of fractions as given in a test. Found a marked degree of difficulty for the three examples which were essentially of the same type. Describes an attempt then made to determine the factors of difficulty.

6. Ilg, Frances and Ames, Louise B., "Developmental Trends in Arithmetic," *Journal of Genetic Psychology* 79: 3–28, September, 1951.

Describes a longitudinal study of developmental trends in arithmetic for some 30 children. Findings suggest discrimination between oral and written experience, importance of knowing processes used by individual children, determination of "time-tables" from what a child is able to do, and differences between boys and girls in awareness and interest in number concepts.

7. Harding, Lowry W. and Bryant, Inez P., "An Experimental Comparison of Drill and Direct Experience in Arithmetic Learning in a Fourth Grade," *Journal of Educational Research* 37: 321–337, January, 1944.

Reports a study which compares two methods of teaching arithmetic by one teacher to two groups of children. As measured by the reasoning ability of achievement tests, the direct-first hand experiences method was more effective than the method using drill procedures and vicarious experiences.

8. Holmes, Darrell, "An Experiment in Learning Number Systems," *Educational Research Bulletin* 28: 100–104; 111–112, April, 1949.

Describes a study with 7th grade children in learning different number systems. Found that these children could learn different number systems and that such learning aided in the understanding of the decimal system.

9. McAllister, Brigid, "Arithmetical Concepts and the Ability to Do Arithmetic," *British Journal of Educational Psychology* 21: 155–156, June, 1951.

Summary of an investigation made in two primary classes to find the relationship between the development of arithmetical concepts and specific and general intellectual ability. Found that various aspects of arithmetic are dependent on both an ability specific to arithmetic and also a general factor designated as an aspect of general intelligence.

10. MacLatchy, Josephine H., "A Test of the Pre-School Child's Familiarity With Measurement," *Educational Research Bulletin* 29: 207–208; 222–223, November, 1950.

Describes an inventory test to be used with six year olds in finding out what they know about measurement when they enter school. Gives some findings and the test is offered to other teachers interested in working together to find out what large numbers of pre-school and first grade children know about measurement.

11. Mott, Sina M., "Number Concepts of Small Children," *The Mathematics Teacher* 38: 291–301, November, 1945.

Reports an investigation made of number concepts of 4 and 5 year old children attending kindergarten and nursery school for purposes of studying the development of children's number concepts. Findings are summarized with regard to ability to do rote counting, knowledge of number names, ability to count objects, concepts of numbers 1–10, ordinal number, application of the processes of addition and subtraction, and the use of the other arithmetical concepts.

12. Mott, Sina and Martin, Mary Elizabeth, "Do First Graders Retain Number Concepts Learned in Kindergarten?" *The Mathematics Teacher* 40: 75–78, February, 1947.

Describes an investigation (a follow-up of #11) to determine the number concepts retained by children going from kindergarten to first grade. Found that, with the exception of counting by rote from 1 to 100, children carry over into the first grade the number experiences which they learned in kindergarten.

13. Mulholland, Vernie, "Fifth Grade Children Discover Fractions," *School Science and Mathematics* 54: 13–30, January, 1954.

Describes a study made with fifth grade children in testing the hypothesis—given suitable materials, children make discoveries of mathematical relationships on their own. Includes descriptive lessons used in the experiment and the evaluation of the success of the method.

14. Plank, Emma N., "Observations on Attitudes of Young Children Toward Mathematics," *The Mathematics Teacher* 43: 252–263, October, 1950.

Reports observations made of a selected group of children who were having difficulty with arithmetic. Found achievement more strongly related with problems of personal adjustment than with intelligence or school experience. The use of Montessori materials not only established rapport with these children but also relaxed them and, furthermore, challenged their thinking.

15. Ramharter, Hazel K. and Johnson, Harry C., "Method of Attack Used by 'Good' and 'Poor' Achievers in Attempting to Correct Errors in Six Types of Subtraction Involving Fractions," *Journal of Educational Research* 42: 586–597, April, 1949.

Reports a study made to determine the differences, if any, in the methods of attacks used by "good" and "poor" achievers in attempting to correct errors in six types of subtraction involving common fractions. Found that good achievers had greater evidence of insight, were more disgusted with previous errors, made more efficient use of a guide sheet, and used greater ingenuity in proving examples to eliminate errors than did the "poor" achievers.

16. Semmelmeyer, Madeline, "External Methods in Dealing With Abstractions in Reading," *Elementary School Journal* 50: 28–36, September, 1949.

Describes an experiment in introducing the subject of area in an eighth grade class, testing the hypothesis, "A child must acquire meaning through his own active responses to his environment." Implication was drawn that meaning of abstract terms can be developed through the application of extensional methodology.

17. Spache, George, "A Test of Abilities in Arithmetic Reasoning," *Elementary School Journal* 47: 442–445, April, 1947.

Reports the construction of an arithmetic reasoning test in attempting to measure not only the actual solution of the problem but also the pupil's ability to recognize or understand the facts given, to identify the problem, to choose the appropriate arithmetic computation, to estimate a probable solution, and to execute the final solution. Found ample reason to believe that some reading skills are related to achievement in problem solving.

18. Stot' r, Carolyn, "Arithmetic Concepts of Preschool Children," *Eelmentary School Journal* 46: 342–345, February, 1946.

Presents some results from an investigation designed to determine certain facts concerning the ability of young children to make progress in developing an understanding of numbers before they enter school. Concluded that children of ages from 50–69 months have a definite consciousness and an understanding of numbers before they enter school.

ARITHMETIC RESULTS IN 1930 AND IN 1954

We hear claims and counterclaims about the product of our public schools. On the one hand, business men tell us that high school graduates don't seem to know how to attack a simple mathematical exercise and can't even add and similarly teachers of college freshmen say that their students can't reason and perform operations very uncertainly. On the other hand, some educators cite "researches" which "prove" that pupils today outrank those of 25 years ago. Just what is the situation?

Instead of a statistical comparison of total scores on an arithmetic test we probably should be seeking information such as: (a) in what rsepects is the learning of pupils today better or poorer than it was 25 years ago? and (b) how well do our students perform in relation to the demands of society upon them? What are the strengths and the weaknesses and what should be done about this?

BIBLIOGRAPHY FOR TEACHERS

Brueckner, Leo J. and Foster E. Grossnickle, *How to Make Arithmetic Meaningful.* Philadelphia, The John C. Winston Co., 1947, 513 p., $4.00.

Buckingham, Burdett R., *Elementary Arithmetic, Its Meaning and Practice.* Boston, Ginn and Co., 1947, 744 p., $5.50.

Central New York School Study Council, *Developing Meaningful Practices in Arithmetic.* Syracuse, The Council, 1951, 123 p., $2.00.

Clark, John R., and others, *Primary Arithmetic Through Experience.* Yonkers, N. Y., World Book Co., 1953, $2.72.

Eves, Howard, *An Introduction to the History of Mathematics.* New York, Rinehart and Co., Inc., 1953, 422 p., $6.00.

Glennon, Vincent and C. W. Hunnicutt, *What Does Research Say About Arithmetic?* Washington, Association for Supervision and Curriculum Development, NEA, 1952, 45 p., $.50.

Harding, Lowry W., *Functional Arithmetic: Photographic Interpretation.* Dubuque, Iowa, Wm. C. Brown Co., 1952, 204 p., $2.00.

Hickerson, James A., *Guiding Children's Arithmetic Experiences.* New York, Prentice-Hall, Inc., 1952, 322 p., $6.65.

Morton, Robert L., *Teaching Children Arithmetic.* New York, Silver Burdett Co., 1953, 566 p., $4.50.

National Council of Teachers of Mathematics, *Arithmetic in General Education.* Sixteenth Yearbook. New York, Bureau of Publications, Teachers College, Columbia University, 1941, 335 p., $1.25.

National Council of Teachers of Mathematics, *A Source Book of Mathematical Applications.* Seventeenth Yearbook. New York, Bureau of Publications, Teachers College, Columbia University, 1942, 291 p., $2.00.

National Council of Teachers of Mathematics, *The Learning of Mathematics, Its Theory and Practice.* Twenty-first Yearbook. Washington, The Council, 1953, 355 p., $4.00.

National Society for the Study of Education, *The Teaching of Arithmetic.* Fiftieth Yearbook, Part II. Chicago, University of Chicago Press, 1951, 302 p., $3.50.

Rosenquist, Lucy L., *Young Children Learn to Use Arithmetic.* Boston, Ginn and Co., 1949, 174 p., $2.75.

Sanford, Vera, *A Short History of Mathematics.* Boston, Houghton Mifflin Co., C 1930, 402 p., $4.50.

Smith, David E., *History of Mathematics.* Vol. 2. Boston, Ginn and Co., C 1925, 725 p., $6.50.

Smith, David E. and Jekuthiel Ginsburg, *Numbers and Numerals; A Story Book for Young and Old.* New York, Bureau of Publications, Teachers College, Columbia University, C 1937, 52 p., $.35.

Spencer, Peter L. and Marguerite Brydegaard, *Building Mathematical Concepts in the Elementary School.* New York, Henry Holt and Co., 1952, 372 p., $4.00.

Spitzer, Herbert F., *The Teaching of Arithmetic.* Boston, Houghton Mifflin Co., 1948, 397 p., $3.25.

Stern, Catherine, *Children Discover Arithmetic.* New York, Harper and Brothers, 1949, 295 p., $4.50.

Stokes, Claude N., *Teaching the Meanings of Arithmetic.* New York, Appleton-Century-Crofts, Inc., 1951, 531 p., $4.50.

Wheat, Harry G., *How to Teach Arithmetic.* Evanston, Ill., Row, Peterson and Co., 1951, 438 p., $3.00.

History of Mathematics in Teaching Arithmetic

MARGARET F. WILLERDING
Harris Teachers College, St. Louis

LEARNING ABOUT THE HISTORICAL development of some phases of arithmetic not only serves as a basis for better understanding of our civilization but also aids in creating a favorable attitude and motivation for learning. Many teachers, because of their lack of knowledge, overlook the history of mathematics as a source of enrichment in teaching arithmetic. The development of our number system, of fractions, and units of measurement is as exciting to many pupils as the accounts of wars and other political conflicts in the struggle for freedom. In fact, modern society is very dependent upon number and quantity and the ways in which these are interpreted and used.

Reference books such as *Sanford* and *Smith and Ginsburg* listed in the Bibliography for Teachers in this issue have chapters dealing with topics in arithmetic. Some of the topics which an elementary school teacher might use are: counting; numbers, numerals, and number system; time and calendar; common and decimal fractions; weights and measures; and number mysteries and puzzle problems. Encyclopedias, especially those written for children, are another source of background information.

Stories of number development interest and fascinate both children and adults. Bringing to a class this information at the opportune time stimulates interest in numbers and in the ways we work with them. It also adds to the understanding and appreciation of the development of our civilization. For example, the modern use of zero, the decimal point, ways of writing fractions, our changing uses of common measures, the struggle for standardized measures, and the comparatively recent agreement on "standard time" are a few samples of historical topics.

Another type of historical material that is useful in intermediate and upper grades is the investigation of older methods of computation. These may range from uses of the abacus and other counting frames to "scratch methods" and to modern calculators and electronic computers. To boys particularly these machines and devices are appealing.

There are many ways to relate the history of mathematics to the curriculum. Here too is a fine opportunity to correlate other fields with arithmetic. Methods of presentation include: stories, bulletin boards, dramatizations, assembly programs, murals, notebooks, and visual aids. The story and the drama seem best suited for lower grades. Bulletin boards, murals, and dramatizations have been successful in intermediate grades. A display of pupil-made devices or an assembly program may include all types of materials such as "finger reckoning," calculation on a counter, "scratch subtraction," old and modern units of measure, etc. There is almost no limit to what a resourceful teacher may do. She will of course always be aiming for worthwhile learning as an outgrowth of what may, for the moment, seem to be amusement.

Many stories and simple dramatizations can be prepared on the themes of counting and numeration. Sticks, stones, fingers, and similar objects are useful in representing the counts 1, 2, 3, etc. in a fashion similar to that which primitive man probably used. Then another object may be used to represent ten as ten *one's* are exchanged for one *ten*. Thus the principle of *place value* and the *base ten* are illustrated. This not only provides interesting drama but it lays a sound base for meaningful learning. Similarly, the marks or tallies which children make for keeping scores in games are basically the same idea as various forms of tally reckoning formerly used.

A bulletin board featuring various ways of writing the numerals will interest chil-

dren. This may include the development of our Hindu-Arabic symbols as illustrated in *Smith and Ginsburg* or the ways that ancient civilizations such as the Roman, Greek, Egyptian, and Babylonian made number symbols. This might include speculation about reasons why one group used papyrus while another used clay tablets.

One class may prepare a pantomime or other type of program to be presented before other groups. The story of the development of systems of weights and measures has been successfully used for this type of program. Such units as foot, inch, stone, etc., when given in terms of their probable origin, take on new significance and children tend to remember better those things in which they have had a share in preparation. Another type of program may be based upon explanation of some of the common numerical measures used to classify goods, as for example, thread size, grades of sandpaper, shoe sizes, etc. The central theme of a program based upon measurement may be the development from crude and unstandardized units to modern units and precise measurement. This in fact is the story of our economic and industrial advance of the past few centuries.

Puzzle problems and magic squares such as those illustrated on page six of this issue intrigue both young and old. A simple problem or magic square can be introduced as low as grade four. Not only do these exercises stimulate interest but they also can be used to teach a good deal of arithmetic.

The sugestions given here are a mere sample of the wealth of material that can be used in presenting the history of mathematics as a motivating factor and as common culture in the teaching of arithmetic. Many schools have found this to be a field of exploration for the more gifted children. Some of the notebooks these pupils have prepared show a level of understanding and appreciation far above the chronological age of the pupils. The teacher should be able to help pupils find sources of information and to give a lead and impetus to such work which develops resourcefulness and frequently stimulates the pupil into a worthwhile adult vocation. Every school should have a good encyclopedia and also a reference book in the history of mathematics. In selecting a reference book it is usually wise to choose one that is topically arranged such as *Sanford* or *Smith. Buckingham* also has a good deal of historical information.

"HOW WE DO IT"—YOUR HELP PLEASE

In recent years a number of schools have prepared interest-supplements to their courses of study. These tend to take the form of suggestions for teaching certain phases of arithmetic or for carrying out projects related thereto. Usually a committee of teachers has pooled the experiences of a staff and thus these "How We Do It" booklets represent the combined experience of many people. A county in Virginia has a guide for relating the school and out-of-school experiences of children. A school in California has mimeographed procedures for teaching fractions meaningfully. The alert teacher is always looking for new ideas which will help to make learning more meaningful and to insure its achievement.

Although some of these materials are copyrighted to prevent their commercial exploitation, most of them are available to teachers. THE ARITHMETIC TEACHER wishes to publish a list of materials so that other schools may benefit from seeing what some schools have done. To do this the editors need the help and cooperation of schools having already prepared various "How We Do It" items. Specimen sets of materials that are available to other schools for purchase, for free, or for loan should be sent to Prof. Esther Swenson, Associate Editor with the form printed below.

To Prof. Esther Swenson,
School of Education
University of Alabama
University, Ala.

Enclosed are the following materials dealing with the teaching of arithmetic. These are available as shown below.

Titles of booklets or materials	Grade levels	Prices
_____	_____	_____
_____	_____	_____
_____	_____	_____

The above materials are available:

_____ by request from teachers _____ for loan only

Name and address to be used in sending for materials:

_____

_____

This report was prepared by _____

address _____

SUMMER INSTITUTES AND WORKSHOPS

Teachers College, Columbia University, July 12-16

A special workshop in Teaching Arithmetic featuring topics such as building concepts, using materials and experiences, rationalizing computations, developing problem solving, and evaluating learning. Staff includes Howard F. Fehr, George H. McMeen, and Ben A. Sueltz. For information write to Prof. Howard F. Fehr at Teachers College.

The University of Wisconsin, July 12-13

A two-day conference on Teaching Arithmetic will include papers on research and recent investigations by national leaders. Discussions based upon the presentations will be of interest to teachers, supervisors, and principals. The conference is sponsored jointly by the University and the Wisconsin Mathematics Council. Requests for additional information should be addressed to Professor John R. Mayor, Education Building, Madison 6, Wisconsin.

The arithmetic conference at Wisconsin will be followed by a two-day conference on teaching mathematics in the secondary schools.

OUR NUMBER SYSTEM AND ITS BASE

The Hindu-Arabic system of numbers which we use is regarded by many as the greatest basic achievement of civilized man. With ten symbols, 0, 1, 2, · · · 9, and the principle of *place value* we can write numbers indefinitely. If someone writes any number, regardless how large, this number becomes 10 times as much merely by annexing a zero at the right and thus moving all the digits one place to the left, the next higher order. We call it a decimal system because it uses the base 10, e.g., the number 3046 represents $(6\times10^0)^*+(4\times10^1)+(0\times10^2)+(3\times10^3)$. Similarly the 10's base may be extended to the right of a decimal point, e.g., the number .275 represents $(2\times1/10^1)+(7\times1/10^2)+(5\times1/10^3)$.

The decimal base (10) probably is due to our possession of 10 fingers or digits. But is it a good base? Would 8 or 12 serve better? Any base may be used with place value or positional notation: the number 235 using base 8 means $(5\times8^0)^*+(3\times8^1)+(2\times8^2)$ which in our 10's system equals $5+24+128$ or 157. This same 235 on the base 12 equals 329 on the base 10. Are there any advantages in base 8 or base 12? Consider the fractions below and their relative simplicity in the positional notation of three different bases.

	Base 8	Base 10	Base 12
1/2	4/8 = .4	5/10 = .5	6/12 = .6
1/4	2/8 = .2	2.5 10 = .25	3/12 = .3
1/8	1/8 = .1	1.25/10 = .125	1⅙/12 = .16
5/8	5/8 = .5	6.25/10 = .625	7½/12 = .76

The fraction series of halves, fourths, and eighths constitutes the most commonly used fractions. Note how simple they are in positional notation of base 8. Such numbers are easier to use in paper and pencil figuring and they can be set directly on a computing machine.

Consider the 12 inches in a foot and 12 things in a dozen. If we used a base 12 number system, then 6 ft. 8 in. could be written 6.8 ft. and such numbers could be used in computations like we use decimal fractions. Certainly this would simplify work with perimeters and areas. But a base 12 would require two additional symbols, perhaps *t* for our ten and *e* for our eleven.

In our base 10 system pupils must learn 100 number combinations of addition, subtraction, and multiplication. But on base this would be reduced to 64 with 7×7 as the largest. On the base 12 we would learn 144 basic combinations in each of these three operations. The new electronic computers use a base 2 in which the largest addition combination is $1+1=10$. Of course base 2 numbers appear very large, they use many places, but a machine that performs thousands of operations per minute is not worried by time and size.

We will probably not consider changing the base of our Hindu-Arabic system but we might well consider sectioning more of our common measures decimally instead of into 12ths, 16ths, etc. Thus we could more readily take advantgae of positional notation in computing with paper and pencil and on machines. Occasionally one finds a pupil in grade 7 or 8 who is inquisitive and speculates on the nature of things. Such a pupil might explore number systems and number bases. This is a topic of tremendous ramifications, one that might stimulate the mind of a future "inventive thinker." Let us not overlook him.

* Any quantity having a zero exponent takes a value of one.

NUMBERS AT WORK

by

Patton and Young

A new arithmetic series—copyright 1952-53. Topflight authorship, sound research, proven methods, combined with modern approach and colorful format makes this series the leader in the field.

●

IROQUOIS PUBLISHING COMPANY, INC.

Home Office: **Iroquois Building**
Syracuse, New York

New York Chicago Atlanta
Dallas

THE SLOW LEARNER IN MATHEMATICS

We all encounter pupils whom we describe as "slow learners." Books on psychology provide some general information about such pupils but we lack specific advice on how they learn and how they should be taught. The questions below deal with several facets of the problem. Is there a group of teachers in the United States who will pool their information and experience and write a good article which answers these and other questions dealing with the slow learner?

1. How can we readily recognize the pupil who is a genuine slow learner and not just one who is indolent, uninterested, or lazy? What are his characteristics: of mind, of social and emotional development, and of physical maturity? We need more than the score on an I.Q. test.
2. Does he use the same mental processes as the rapid learner except that he takes longer in the process? Does he have a different characteristic development and mode of thinking? Do slow learners tend to fall into a pattern or do they differ widely in their mental processes? Is it true that those who achieve slowly remember longer?
3. Are there limits in the amount and the complexity of the things which these people can learn? What kinds of things are most readily achieved by these learners? What other things can they learn with surety even though with difficulty?
4. Is it better for the slow learner to learn fewer things very well rather than to be uncertain about many things? What are reasonable goals for these people?
5. What methods of teaching and learning are better suited to the slow learner? Does he need more drill or is the apparent need for drill merely a quick but wrong answer to his problem of understanding?